MATH 1650: SECTION 3.3: RATIONAL FUNCTION INEQUALITIES

To solve an inequality involving a rational function, we use a Sign Diagram.

EXAMPLE: Solve $\dfrac{x^2-2x+1}{x+5} \le 0$. Write your answer using interval notation.

Letting $f(x) = \dfrac{x^2-2x+1}{x+5}$, we use a Sign Diagram to solve $f(x) \le 0$.

We first find values excluded from the domain of f. We solve $x+5=0$ so $x=-5$.

Next, we find the zeros of f by solving $f(x) = \dfrac{x^2-2x+1}{x+5} = 0$.

We get $x^2-2x+1=0$ or $(x-1)^2=0$. Hence, the only zero of f is $x=1$.

Our Sign Diagram is:

$$\begin{array}{ccccccc} (-) & ? & (+) & 0 & (+) & f(x) \\ \longleftarrow & -5 & \text{———} & 1 & \longrightarrow & x \end{array}$$

Hence, $f(x) \le 0$ on $(-\infty,-5) \cup \{1\}$.

EXAMPLE: Solve $\dfrac{5-x}{x^2-x-2} \ge 0$. Write your answer using interval notation.

EXAMPLE: Solve $\dfrac{2x-1}{x+3} \geq 4$. Write your answer using interval notation.

We rewrite the inequality so as to compare with 0 and get a common denominator:

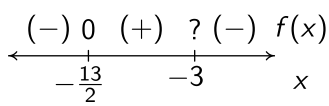
$$\begin{aligned}
\frac{2x-1}{x+3} - 4 &\geq 0 \\
\frac{2x-1}{x+3} - \frac{4}{1} &\geq 0 \\
\frac{2x-1}{x+3} - \frac{4(x+3)}{1(x+3)} &\geq 0 \\
\frac{2x-1}{x+3} - \frac{4x+12}{x+3} &\geq 0 \\
\frac{(2x-1)-(4x+12)}{x+3} &\geq 0 \\
\frac{-2x-13}{x+3} &\geq 0
\end{aligned}$$

We let $f(x) = \dfrac{-2x-13}{x+3}$ and make a Sign Diagram to solve $f(x) \geq 0$.

To find the values excluded from the domain of f, we solve $x+3=0$ so $x=-3$.

To find the zeros of f we solve $f(x) = \dfrac{-2x-13}{x+3} = 0$. We get $-2x-13=0$ so $x = -\frac{13}{2}$.

Our Sign Diagram for $f(x)$ is below.



Solving $f(x) \geq 0$, we get $\left[-\frac{13}{2}, -3\right)$.

EXAMPLE: Solve $\dfrac{x+1}{2x-5} \leq 1$. Write your answer using interval notation.

MATH 1650 APPLICATIONS OF RATIONAL FUNCTIONS

EXAMPLE: The cost $C(x)$ in millions of dollars to seize $x\,\%$ of the illegal drug ‘Shadow’ is given by:

$$C(x) = \frac{300x}{100 - x}, \; 0 \le x < 100$$

- Find and interpret $C(20)$.

- How much Shadow could be seized if \$100 million were spent on its removal?

- Find and interpret the vertical asymptote to the graph of $y = C(x)$.

DEFINITION: If $C(x)$ is the cost of producing x items, the **average cost per item**, denoted $\overline{C}(x)$ is given by:

$$\overline{C}(x) = \frac{C(x)}{x}, \quad x > 0$$

EXAMPLE: The cost, in dollars, to produce x "I'd rather be a Sasquatch" T-Shirts is $C(x) = 2x + 26$, $x \geq 0$.

- Find an expression for $\overline{C}(x)$.

- Find and interpret $\overline{C}(20)$.

- Solve and interpret $\overline{C}(x) = 3$.

- Find and interpret the horizontal asymptote of the graph of $y = \overline{C}(x)$.

HOMEWORK: Section 3.3: 1 - 35 odd